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Wigner trajectories in resonant-tunneling diodes under transverse magnetic field

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We generalize the Wigner formalism to include effects of an in-plane magnetic field and present theoretical study of effects of a transverse magnetic field on tunneling dynamics for resonant-tunneling diodes. We solve transport equation for the steady-state Wigner distribution function. Particle trajectories are determined in phase space as contour lines of the distribution function. From trajectories we estimate tunneling times. Discussions of magnetic field dependence and transverse momentum dependence of tunneling times are presented.

I. INTRODUCTION

The present availability of high-quality semiconductor heterostructures and high magnetic fields has stimulated many experimental studies of magnetotransport in low-dimensional electronic systems. Both theoretical and experimental works have rapidly accumulated, in particular, in the area where effects of a transverse magnetic field on electron tunneling through a heterostructure are studied.[1–9,18,19] In a transverse magnetic field, electrons execute cyclotron motion while tunneling through barriers in the plane perpendicular to the field. An interesting topic arises regarding effects of cyclotron motion on dynamics of electron tunneling. Study of this subject is not only important in its own right, but also provides more information on dynamical aspects of electron tunneling than a similar study but with lack of the magnetic field.

Recently, a quantum theory of electron transport in the Wigner formalism has been developed and applied to resonant-tunneling structures.[10–13] It is free of the major criticism of earlier tunneling theories using the Wentzel–Kramers–Brillouin (WKB) approximation[14] or transfer matrices,[15] both of which require knowledge of the distribution of electrons at each side of the tunneling interface, rather than the bulklike distribution far from the tunneling interface, although the latter is usually used.[12] Furthermore, it offers the possibility of describing quantum-mechanical results in terms of classical pictures such as particle trajectories.[13,16]

In this article, we generalize the Wigner formalism and report theoretical study of effects of a transverse magnetic field on particle trajectories and tunneling times for a resonant-tunneling diode made of GaAs and AlGaAs. In Sec. II, we present the theoretical model. In Sec. III, we present the results. In Sec. IV, we summary our study.

II. THEORETICAL MODEL

Quantum theory of electron transport in the Wigner formalism can be generalized to include effects of an in-plane magnetic field. The generalized theory will be detailed elsewhere. We sketch it only briefly in this paper. We consider a multilayer structure with growth direction along z axis. The device active length is taken to be l. The structure is connected to reservoirs at both sides, which are assumed to have properties analogous to those of a black body.[10] The magnetic field is taken to be along x axis with $\mathbf{B}=(B,0,0)$. The field is assumed to be confined in between the two outmost interfaces of the structure located at $z=a$ and at $z=b$ with $a<b$. The assumption of confinement is made because, in usual experimental situations, the electrodes are heavily doped and impurity scattering prevents cyclotron motion of electrode electrons. For the corresponding vector potential, we use a particular gauge in which $\mathbf{A}=(0,-Ba,0)$ for $z<a$, $\mathbf{A}=(0,-Bz,0)$ for $a<z<b$, and $\mathbf{A}=(0,-Bb,0)$ for $z>b$. We consider electron transport in an n-type GaAs/GaAlAs heterostructure in particular. Only tunneling via conduction band needs to be treated. We neglect spin–magnetic field interaction and write the Hamiltonian

$$H=\frac{P_x^2}{2m^*}+\frac{(P_y+eA_y)^2}{2m^*}+\frac{P_z^2}{2m^*}+v(z) \tag{1}$$

within effective-mass approximation. Here, m^* is the conduction band effective mass, and $v(z)$ represents potential profile including effects of conduction band offset. The Hamiltonian has translational symmetry in both x and y directions. Hence, k_x and k_y serve as good quantum numbers labeling the wave function of a tunneling electron. Here, $\hbar k_x$ is the mechanical momentum along x axis and $\hbar k_y$ is the canonical momentum along y axis. For the mechanical momentum along y axis $\hbar k_y^{\text{mech}}$, $(\hbar k_y+eA_y)$ is to be used. We define the generalized Wigner distribution function $f(Z,k_z,t;k_x,k_y)$, which represents the density of particles in phase volume $dZ\,dk_z$ with quantum numbers

k_x and k_y at time t. The transport equation, together with boundary conditions, for the distribution function is

$$\frac{\partial}{\partial t}f(Z,k_z,t;k_x,k_y)=-\frac{\hbar k_z}{m^*}\frac{\partial}{\partial Z}f(Z,k_z,t;k_x,k_y)$$

$$-\frac{1}{\hbar}\int\frac{dk_z'}{2\pi}V(Z,k_z-k_z';k_y)$$

$$\times f(Z,k_z',t;k_x,k_y), \qquad (2)$$

$$f(0,k_z,t;k_x,k_y)=f_l(k_x,k_y-eBa/\hbar,k_z), \quad k_z>0,$$

$$f(l,k_z,t;k_x,k_y)=f_r(k_x,k_y-eBb/\hbar,k_z), \quad k_z<0,$$

where f_l and f_r are equilibrium distribution functions of the left-hand and right-hand reservoirs, respectively. Here, the kernel of the potential operator $V(Z,k_z;k_y) = 2\int_0^\infty d\xi \sin(k_z\xi)[v_t(Z+\xi/2) - v_t(Z-\xi/2)]$ with $v_t(z) = v_{\mathrm{offset}}(z) + v_{\mathrm{ext}}(z) + e\hbar k_yA_y(z)/m^* + e^2A_y^2(z)/2m^*$ containing the conduction band offset, external electrical potential, and magnetic effects.

To solve Eq. (2) numerically for steady-state solution, we replace the continuous domain of the problem by a lattice of discrete points in phase space. $\partial f/\partial Z$ is approximated by a second-order difference expression. We apply Gaussian elimination to the discretized equation and solve for $f(Z,k_z;k_x,k_y)$. With $f(Z,k_z;k_x,k_y)$, we can determine trajectories by finding the contour lines along which the function $f(Z,k_z;k_x,k_y)$ is constant. It is possible to define a tunneling time[13] for those trajectories which traverse the barriers, using the formula which is valid classically $t(Z) = \int^Z dZ'\, m^*/\hbar k_z(Z')$.

III. RESULTS

We apply the theory to a resonant-tunneling diode $GaAs/Al_{0.3}Ga_{0.7}As/GaAs/Al_{0.3}Ga_{0.7}As/GaAs$ at zero bias. The magnetic field is taken to be along $-x$ axis. The GaAs well is 50 Å wide. The $Al_{0.3}Ga_{0.7}As$ layer is 28 Å wide. The doping level at both GaAs electrodes is $2\times10^{18}/cm^3$. The temperature is taken to be 300 K. The device active length is taken to be 451 Å. The conduction band effective mass, for simplicity, is taken to be 0.067 in units of free electron mass throughout the device. The conduction band offset is taken to be 0.3 eV.[17] The origin of Z is located at the leftmost interface of the device. With this choice of origin, the canonical k_y is equal to the mechanical k_y^{mech} of electrons at the leftmost interface.

In Fig. 1, we present a few Wigner trajectories. The magnetic field is taken to be 10 T. Both $k_x$and k_y are taken to be zero. The trajectories presented here are all open in the sense that they begin at one side and thread through to the other side of the device. They represent the motion of electrons emitted into the device from the left reservoir. With $k_z \approx 0.56$ nm^{-1} far from the interface, the total energy of the incident electron is about 0.18 eV, and is lower than the barrier height. The trajectories are not symmetric with respect to the middle of the well due to magnetic effects. Electrons decelerate inside the barriers and accelerate in the well as expected. Furthermore, the electrons execute cyclotron motion while tunneling through the structure, with k_z constantly modulated by the magnetic field. Note the decrease in k_z in the well near the right barrier as shown in the plot. In particular, the downmost trajectory nearly hits the $k_z=0$ axis, implying that the corresponding tunneling electron is slowed down by the magnetic field to nearly a halt. This is in agreement with the usual description of transverse magnetic effects as effectively increasing the barrier height.[3]

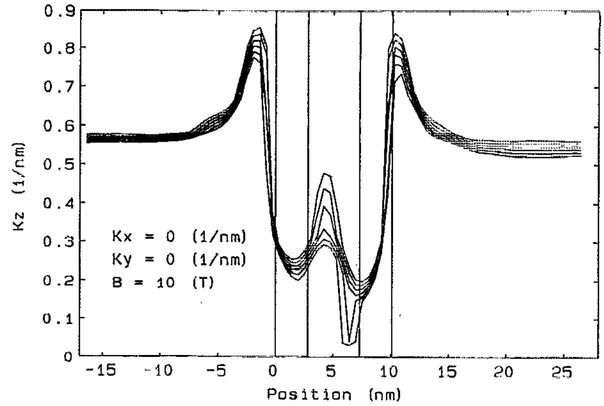

FIG. 1. Wigner trajectories. Both k_x and k_y are taken to be zero. The magnetic field is taken to be 10 T. The middle region bounded by two vertical lines is the well of the structure. The regions next to it are the two barriers.

In Fig. 2, we present $t(Z)$ for the open trajectories shown in Fig. 1. In the plot the inverse slope of $t(Z)$ is equal to the electron velocity v_z. As shown here, the tunneling process is slowed down as the electron traverses through the barriers. In particular, the uppermost curve arises rather steeply as the electron approaches the second barrier. This curve corresponds to the downmost trajectory in Fig. 1 and therefore the rapid time increase can be attributed to the fact that k_z of the electron continuously

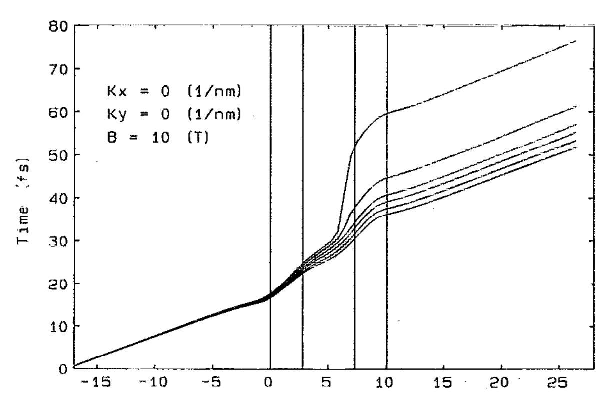

FIG. 2. $t(Z)$ for the open trajectories shown in Fig. 1. The time t is taken to be zero when electrons just enter the device. The middle region bounded by two vertical lines is the well of the structure. The regions next to it are the two barriers.

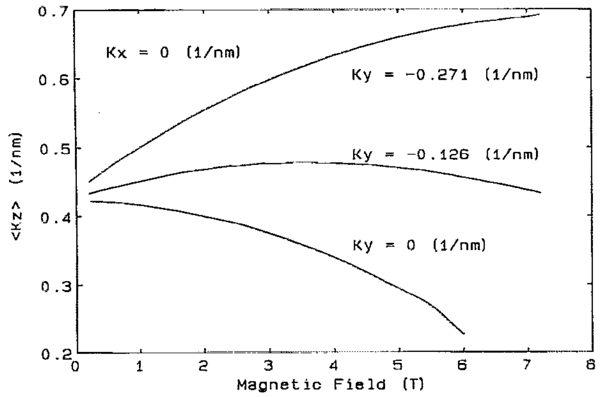

FIG. 3. $\langle k_z \rangle$ vs magnetic field, where $\langle k_z \rangle$ is the value of k_z along a trajectory averaged over the well region. k_x is taken to be zero. Three values of k_y are considered, namely, -0.271, -0.126, and 0 nm^{-1}.

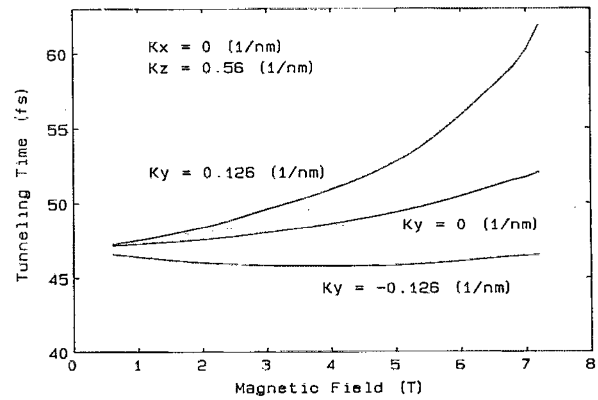

FIG. 4. Tunneling times vs magnetic field. Three values of k_y are considered, namely, -0.126, 0, and 0.126 nm^{-1}, respectively. k_z of the incident electrons is taken to be 0.56 nm^{-1} and k_x is taken to be zero.

decreases during cyclotron motion and the electron moves rather slowly as k_z approaches minimum near the second barrier.

In Fig. 3, we present $\langle k_z \rangle$ versus magnetic field, where $\langle k_z \rangle$ is the value of k_z along a trajectory averaged over the well region. k_x is taken to be zero. Three values of k_y are considered, namely, -0.271, -0.126, and 0 nm^{-1}. As shown here, for fixed magnetic field, $\langle k_z \rangle$ decreases with k_y increasing towards positive values. For the two negative values of k_y considered, $\langle k_z \rangle$ increases at first and eventually decreases with the magnetic field. On the other hand, in the case of $k_y = 0$, $\langle k_z \rangle$ decreases monotonically with the field. They are in agreement with the classical picture of cyclotron motion. Under a magnetic field that is along $-x$ direction, the electron travels in clockwise sense in y–z plane. The Lorentz force induces a change $\Delta k_z = -\int |eB| (k_y + |eB| z/\hbar)/m^* \, dt$ in k_z. For negative values of k_y, this effect increases $\langle k_z \rangle$ at low fields (since the first term in the bracket dominates) and reduces $\langle k_z \rangle$ at high fields (since the second term in the bracket dominates). For nonnegative values of k_y the effect reduces $\langle k_z \rangle$.

In Fig. 4, we present tunneling times versus magnetic field. Three values of k_y are considered, namely, -0.126, 0, and 0.126 nm^{-1}, respectively. k_z of the incident electron is taken to be 0.56 nm^{-1} and k_x is taken to be zero. The three curves in the plot converge to the same value at low magnetic field but become substantially different at high magnetic field. The results for both $k_y = 0$ nm^{-1} and $k_y = 0.126$ nm^{-1} increase with increasing the magnetic field. On the other hand, the tunneling time for $k_y = -0.126$ nm^{-1} decreases at first before arising with the field. It indicates that in this case the magnetic field alters velocity of electrons in such a way as to speed up their tunneling. This is consistent with the magnetic-field dependence of $\langle k_z \rangle$ shown in Fig. 3, where in the case of $k_y = -0.126$ nm^{-1}, $\langle k_z \rangle$ at low field increases with increasing the field.

In Fig. 5, we present tunneling times versus k_y. Three magnetic fields are considered, namely, 0, 4, and 8 T. k_x is taken to be zero and k_z is taken to be 0.56 nm^{-1} before electrons enter the field. The tunneling time is independent of k_y in the case of zero magnetic field. On the other hand, the tunneling times increase with increasing k_y in the presence of magnetic field, due to the fact that $\langle k_z \rangle$ decreases with increasing k_y as shown in Fig. 3. Note, in particular, that when k_y is sufficiently negative the magnetic field reduces the tunneling time in comparison with that at zero field.

IV. SUMMARY

In summary, we have generalized the Wigner formalism to include effects of an in-plane magnetic field. We stress that, although a special gauge for the vector potential has been taken in the theory, the physically interesting quantities such as the tunneling times, the charge distributions, and the currents are gauge invariant and should be independent of our choice of the gauge. We have applied the generalized Wigner formalism to resonant-tunneling diodes under the action of a transverse magnetic field. We have determined particle trajectories in phase space. We

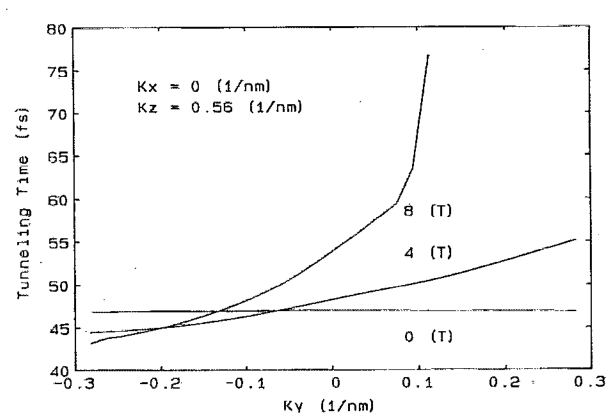

FIG. 5. Tunneling times vs k_y. Three magnetic fields are considered, namely, 0, 4, and 8 T. k_x is taken to be 0 nm^{-1} and k_z of the incident electrons is taken to be 0.56 nm^{-1}.

have calculated tunneling times by integration along each trajectory. This allows for the development of a physical intuition about the actual effects of a transverse magnetic field on tunneling dynamics. It is found that, in general, the magnetic field increases tunneling times, and affects electron tunneling in such a way as though to increase the barrier height. In some cases electrons are even nearly prevented from tunneling through the device by the field. However, depending on the transverse momentum of electrons perpendicular to the magnetic field, cyclotron motion could sometimes speed up tunneling process and reduce tunneling times. Finally, we note that, in our study, although we have excluded in the electrodes skipping orbits[18,19] characteristic of Landau quantization due to the assumption that frequent impurity scattering prevents cyclotron motion, the theoretical method presented in this work can be applied to the case where the electrodes are lightly doped and electrons can execute well-defined skipping orbits.

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